FINAL: BIII REPRESENTATION THEORY

Date: $7^{\mathbf{th}}$ **January 2022**

Class notes may be used for this exam.

The Total points is **44** and the maximum you can score is **40** points.

A **representation** would mean a **representation on a vector space over complex numbers**.

(1) (10 points) Let $\rho : G \to GL(V)$ be a representation. Let χ be the character of V. Let $H = \{g \in G : \chi(g) = \dim(V)\}$. Show that H is a normal subgroup of G. Show that that if G/H is abelian then V is a direct sum of one dimensional subrepresentations.

(2) (12 points) Assuming the order of G is odd, compute the the multiplicities of every irreducible representation of $Ext^2 V$ for V the regular representation of G.

(3) (12 points) Let V be the standard representation of S_3. Let $G = S_3 \times S_3$, H be the subgroup $S_3 \times e$ and D be the subgroup $\{(\sigma, \sigma) : \sigma \in S_3\}$ of G. Note that D and H are isomorphic to S_3. Decompose $W_1 = \mathrm{Ind}_H^G V$ and $W_2 = \mathrm{Ind}_D^G V$ as direct sum of irreducible G-representations.

(4) (10 points) Compute the number of irreducible representations of the alternating group A_7 and dimensions of each of them.